

# DIRAC VS. MAJORANA NEUTRINO MASSES FROM A TEV INTERVAL

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## Abstract

We investigate the nature (Dirac vs. Majorana) and size of left-handed neutrino masses in a supersymmetric five-dimensional model compactified in the interval  $[0, \pi R]$ , where quarks and leptons are localized on the boundaries while the gauge and Higgs sectors propagate in the bulk of the fifth dimension. Supersymmetry is broken by Scherk-Schwarz boundary conditions and electroweak breaking proceeds through radiative corrections. Right-handed neutrinos propagate in the bulk and have a general five-dimensional mass  $M$ , which localizes the zero modes towards one of the boundaries, and arbitrary boundary terms. We have found that for generic boundary terms left-handed neutrinos have Majorana masses. However for specific boundary configurations left-handed neutrinos are Dirac fermions as the theory possesses a conserved global  $U(1)$  symmetry which prevents violation of lepton number. The size of neutrino masses depends on the localization of the zero-modes of right-handed neutrinos and/or the size of the five-dimensional neutrino Yukawa couplings. Left-handed neutrinos in the sub-eV range require either  $MR \sim 10$  or Yukawa couplings  $\sim 10^{-3}R$ , which make the five-dimensional theory perturbative up to its natural cutoff.

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# 1 INTRODUCTION

Within the Standard Model (SM) framework the origin, nature and lightness of neutrinos are still open questions. A commonly accepted explanation for Majorana neutrinos is the so called seesaw mechanism [1, 2]. It requires the existence of a sterile <sup>1</sup> and massive ( $M_R \gg M_Z$ ) right-handed (RH) spinor weakly coupled to the SM left-handed (LH) neutrinos and the Higgs through a Yukawa term yielding thus a light Dirac mass ( $m_D \ll M_R$ ). The lowest mass eigenvalue is then  $m_\nu \sim \frac{m_D^2}{M_R}$  and it is  $\sim meV$  if the Yukawa coupling is weak enough and/or the RH neutrino mass is large enough. It was originally proposed in the context of four-dimensional (4D) models and it predicts the existence of new physics at the scale  $M_R$ . The drawback of the seesaw mechanism is that it either requires an extremely large value of  $M_R$  [out of reach for the Large Hadron Collider (LHC)] or introducing a small value of the 4D Yukawa coupling  $h_\nu \sim 10^{-7}$ . The alternative possibility that neutrinos are Dirac fermions would require much smaller 4D Yukawa couplings  $h_\nu \sim 10^{-14}$ . In any way the nature of neutrinos (Dirac or Majorana), which is essential for many experimental signatures, still remains as the big open question in neutrino physics.

Some of the above problems may find a solution in the context of theories with (compactified) extra dimensions. They provide a natural scale, the inverse compactification radius of extra dimensions  $1/R$ , and moreover they can “naturally” provide very small 4D Yukawa couplings if some of the fields participating in the localized Yukawa interactions are exponentially localized far away. This led to the physical scenario where the RH neutrino belongs to a hypermultiplet propagating in the bulk of the extra dimensions while Yukawa interactions are localized at fixed points of the bulk, e.g. in orbifold compactifications.

In general in a theory defined in an extra dimensional scenario two main points should be answered:

- Why neutrinos are so light?
- What is the nature of neutrinos: Dirac vs. Majorana fermions?

Although lots of different studies have been done so far [3]-[23] in supersymmetric or non-supersymmetric theories, and flat or warped space-time a systematic analysis of the previous

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<sup>1</sup>With respect to the SM gauge group.

questions was not addressed in a compactified theory with arbitrary boundary terms.

In this paper we have investigated in detail the lightness and nature of neutrinos in a supersymmetric five-dimensional (5D) theory defined on the interval  $\mathcal{M}_4 \times [0, \pi R]$  where gauge and Higgs bosons propagate in the bulk of the fifth dimension and SM fields belong to chiral superfields localized in one of the boundaries of the 5D space. We will also consider the compactification scale in the TeV range, which can lead to production of KK excitations [24, 25] at the LHC <sup>2</sup> and where supersymmetry can be globally broken by the Scherk-Schwarz (SS) mechanism [26]. In these theories the supersymmetric spectrum has a very characteristic pattern [27], which can very easily be identified in experiments, and electroweak symmetry breaking (EWSB) can proceed by radiative corrections [29, 30] bypassing some previously outlined difficulties in Ref. [28].

The plan of the paper is as follows. In section 2 the most general supersymmetric action for RH neutrinos in the bulk is presented as well as the boundary terms, and boundary conditions arising from the variational principle are written. In section 3 the wave functions for the bosonic and fermionic components of the RH neutrino hypermultiplet satisfying the equations of motion are deduced and the number of degrees of freedom identified. In section 4 the 4D theory obtained after integrating the fifth dimension is worked out and the effective action for the light neutrino is computed in the background of the Higgs field. Our main physical results can be found in section 5 where the different situations concerning the nature and size of LH neutrino masses are discussed. Finally section 6 contains our conclusions.

A short summary of the main results is appended now. Concerning the nature of the LH neutrino mass we have found that everything depends on the relative orientation of the boundary action (characterized by a vector  $\vec{s}$ ) with respect to the bulk action of the RH neutrino characterized by a mass vector  $\vec{p}$ . In the generic situation where both vectors  $\vec{s}$  and  $\vec{p}$  are arbitrarily oriented, lepton number is violated and we find a Majorana mass for LH neutrinos. In the particular case where  $\vec{s}$  and  $\vec{p}$  are parallel (or anti-parallel) there is a conserved global  $U(1)$  symmetry group which prevents lepton number breaking and produces Dirac masses for neutrinos. As for the size of the LH neutrino masses, in the case of Dirac masses they can be in the meV range if the zero-mode of the RH neutrino is exponentially localized towards the brane opposite to that where the SM fermions localize. In particular if

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<sup>2</sup>This possibility will of course depend on the detailed localization properties of quarks and leptons.

$M$  is the localizing mass of RH neutrinos the correct size for the LH neutrino mass requires that  $MR \sim 10$ . In the generic case where LH neutrino masses are Majorana, in order to get the correct size without imposing anomalously small 5D (dimensional) Yukawa couplings it is shown that quarks and leptons should be localized in opposite boundaries while the bulk Higgs localizes towards the quark boundary. In this case the Yukawa couplings should be somewhat small ( $\sim 10^{-3}R$ ) although for particular values of  $\vec{s} \cdot \vec{p}$  there is a cancellation in the neutrino mass matrix for large values of  $M$  and the correct spectrum of neutrinos can be again obtained for  $MR \sim 10$ .

## 2 THE ACTION AND THE SPECTRUM

The action for the sterile neutrino is that of a 5D  $N = 1$  supersymmetric theory with boundary terms defined in the manifold  $\Sigma = M_4 \times I$ , where  $M_4$  is the Minkowski space and  $I$  is the interval  $[0, \pi R]$ ,  $R$  being the compactification radius that we assume to be  $\lesssim \text{TeV}^{-1}$ . Thus the right handed neutrino belongs to a (SM singlet) hypermultiplet  $(N, N_c)$  with  $N_{(c)} = \phi_{(c)} + \sqrt{2}\theta\psi_{(c)} + \theta^2 F_{(c)}$ .

The explicit form of the action is

$$S = S_{\text{bk}} + S_{\text{bk}}^{\text{m}} + S_{\text{bd}}, \quad (2.1)$$

where

$$S_{\text{bk}} = \int_{\Sigma} d^4\theta [\bar{N}N + \bar{N}_c N_c] - \int_{\Sigma} d^2\theta N_c \partial_5 N + \text{h.c.}, \quad (2.2)$$

$$S_{\text{bk}}^{\text{m}} = \int_{\Sigma} d^2\theta \left[ a N_c N + \frac{b}{2} N^2 - \frac{b^*}{2} N_c^2 \right] + \text{h.c.}, \quad (2.3)$$

$$S_{\text{bd}} = \int_{\partial\Sigma} d^2\theta \left( \frac{\mu}{2} N^2 + \frac{\lambda}{2} N_c^2 + \nu N N_c \right) + \text{h.c.}, \quad (2.4)$$

and  $\mu$ ,  $\lambda$  and  $\nu$  are arbitrary (dimensionless) complex numbers<sup>3</sup>, while the bulk mass terms only depend on two (mass) parameters ( $a \in \mathbb{R}$ ,  $b \in \mathbb{C}$ ) in order to guarantee the  $SU(2)_R$  invariance. In fact the fermionic component of Eq. (2.3) provides the most general fermion mass Lagrangian invariant under the 5D Lorentz transformations

$$a\psi_c\psi + \frac{b}{2}\psi\psi - \frac{b^*}{2}\psi_c\psi_c + \text{h.c.} = a\bar{\Psi}\Psi + \frac{b}{2}\bar{\Psi}\Psi^c + \frac{b^*}{2}\bar{\Psi}^c\Psi \quad (2.5)$$

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<sup>3</sup>It is understood that boundary terms occur independently at  $y = 0$  and  $y = \pi R$ .

where  $\Psi$  is the Dirac spinor

$$\Psi = \begin{pmatrix} \psi_c \\ \bar{\psi} \end{pmatrix} \quad (2.6)$$

$\Psi^c = C_5 \bar{\Psi}^T$  its charge conjugate and  $C_5 = \gamma_2 \gamma_0 \gamma_5$  the 5D charge conjugation operator <sup>4</sup>.

By applying the variation principle on (2.2)-(2.4) we find the boundary conditions

$$\mu N + \nu N_c - N_c = 0, \quad \lambda N_c + \nu N = 0, \quad (2.7)$$

which turn out to be a set of manifestly supersymmetric boundary conditions. One can easily check that the system of equations (2.7) is overdetermined unless

$$\mu \lambda - (\nu - 1)\nu = 0, \quad (2.8)$$

and that it is invariant under the redefinitions

$$N_c \leftrightarrow N, \lambda \leftrightarrow \mu, \nu \leftrightarrow 1 - \nu. \quad (2.9)$$

In the special case  $\nu = 0$  the boundary conditions read

$$\begin{cases} \lambda = 0, \mu N - N_c = 0 \\ \text{or} \\ \mu = 0, N_c = 0 \end{cases}. \quad (2.10)$$

while the case  $\nu = 1$  is obtained from the previous one by means of the relations (2.9). In the general case where  $\nu \neq 0, 1$  Eq. (2.7) reduces to

$$\frac{\lambda}{\nu} N_c + N = 0, \quad (2.11)$$

which means that the complex parameters  $(\nu, \mu, \lambda)$  are highly redundant since only the complex number  $z = \lambda/\nu$  does matter. Notice that by letting  $z$  to take any complex value we cover the whole set of boundary conditions (including  $\nu = 0$ , which corresponds to  $z \rightarrow \infty$ ). Actually one can easily show that the whole complex plane is covered consistently with (2.8) by the restriction

$$\nu \in \mathbb{R}, \quad \mu = -\lambda^*.$$

As a matter of fact a possible parametrization is given by

$$\nu = \frac{1}{2} (1 + s_3), \quad \mu = -\lambda^* = \frac{1}{2} s_-, \quad (2.12)$$

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<sup>4</sup>Note the different definitions of the 5D ( $C_5$ ) and 4D ( $C_4 = -i\gamma_2\gamma_0$ ) charge conjugation operators which gives rise to the minus sign which appears in Eq. (2.3).

with  $s_- = s_1 - is_2$  where  $s_i$  are three real parameters such that  $\vec{s}^2 = 1$ .

The boundary conditions now read as

$$\frac{1}{2}(1 - \vec{s} \cdot \vec{\sigma}) \begin{pmatrix} N_c \\ N \end{pmatrix} = 0, \quad (2.13)$$

where  $\vec{\sigma}$  are the Pauli matrices. In components Eq. (2.13) reads as

$$\begin{aligned} \frac{1}{2}(1 - \vec{s} \cdot \vec{\sigma}) \begin{pmatrix} \phi_c \\ \phi \end{pmatrix} &= 0, \\ \frac{1}{2}(1 + \vec{s} \cdot \vec{\sigma}) [\partial_y + \vec{p} \cdot \vec{s} M] \begin{pmatrix} \phi_c \\ \phi \end{pmatrix} &= 0, \end{aligned} \quad (2.14)$$

$$\frac{1}{2}(1 - \vec{s} \cdot \vec{\sigma}) \begin{pmatrix} \psi_c \\ \psi \end{pmatrix} = 0, \quad (2.15)$$

respectively, where we have defined the shorthands

$$\vec{p} = \frac{1}{\sqrt{a^2 + |b|^2}}(b_R, -b_I, a), \quad M = \sqrt{a^2 + |b|^2}. \quad (2.16)$$

The spectrum allowed by these boundary conditions can be read off from Refs. [29, 30], and it is provided by the zeroes of the function

$$\sin^2(\pi\tau) - (c_0 - c_\pi) \frac{M}{\Omega} \tan(\pi\Omega R) - \left[ \cos^2(\pi\tau) + c_0 c_\pi \frac{M^2}{\Omega^2} \right] \tan^2(\pi\Omega R), \quad (2.17)$$

with  $\cos(2\pi\tau) = \vec{s}_0 \cdot \vec{s}_\pi$ ,  $c_f = \vec{p} \cdot \vec{s}_f$  and  $\Omega^2 = m^2 - M^2$ ,  $m$  being the physical mass eigenvalue. In particular for  $c_0 = c_\pi \equiv c$  ( $\tau = 0$ ) the spectrum predicted by (2.17) is

$$m^2 = s^2 M^2, \quad m_n = \frac{1}{R} \sqrt{M^2 R^2 + n^2}, \quad n = 1, 2, 3, \dots \quad (2.18)$$

with  $s^2 = 1 - c^2$ .

### 3 WAVE FUNCTIONS

In this section we will explicitly write the solutions to the equations of motion and the boundary conditions for the model presented in the previous section. Let us first re-express the action in a more appropriate way. As one can see from (2.3) the most general mass term involves both Dirac ( $a$ ) and Majorana ( $b, b^*$ ) masses and there is a family of continuous transformations

parameterizing the possible mass configurations. These are  $SU(2)$  rotations acting in the space of chiral supermultiplets  $(N_c, N)^T$ <sup>5</sup>. Although those transformations do not leave the action invariant they are symmetries of the spectrum equation (2.17)<sup>6</sup>. According to (2.16) the action (2.1) can be written as

$$\begin{aligned} \mathcal{S} &= \int_{\Sigma} \bar{\mathcal{N}} \mathcal{N}|_{\bar{\theta}^2 \theta^2} - \frac{1}{2} M \mathcal{N}^T \epsilon \vec{p} \cdot \vec{\sigma} \mathcal{N}|_{\theta^2} - \frac{1}{2} \mathcal{N}^T \epsilon \mathcal{N}'|_{\theta^2} + \text{h.c.} \\ &+ \frac{1}{4} \int_{\partial \Sigma} \mathcal{N}^T \epsilon (1 - \vec{s} \cdot \vec{\sigma}) \mathcal{N}|_{\theta^2} + \text{h.c.}, \end{aligned} \quad (3.1)$$

where

$$\mathcal{N} = \begin{pmatrix} N_c \\ N \end{pmatrix}, \quad (3.2)$$

and  $\epsilon$  is the totally antisymmetric 2-tensor defined as

$$\epsilon \equiv \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = i\sigma_2. \quad (3.3)$$

For simplicity we will be interested in a pure Dirac bulk mass term, i.e.  $\vec{p} = (0, 0, 1)$ . The set of unitary rotations bringing  $\vec{p}$  to  $(0, 0, 1)$  is given by  $U(\vec{p})$  where

$$U(\vec{p}) = \frac{1}{2\sqrt{1-s_{\vec{p}}}} \begin{pmatrix} e^{i\delta_{\vec{p}}} & 0 \\ 0 & e^{-i\delta_{\vec{p}}} \end{pmatrix} \begin{pmatrix} 1-s_{\vec{p}}+c_{\vec{p}} & 1-c_{\vec{p}}-s_{\vec{p}} \\ -1+c_{\vec{p}}+s_{\vec{p}} & 1+c_{\vec{p}}-s_{\vec{p}} \end{pmatrix}, \quad (3.4)$$

with  $c_{\vec{p}} = p_3$ ,  $s_{\vec{p}} = \sqrt{1-(p_3)^2}$  and

$$e^{i\delta_{\vec{p}}} = (2s_{\vec{p}})^{-1/2} \left( \sqrt{p_1 + s_{\vec{p}}} - i \frac{p_2}{|p_2|} \sqrt{-p_1 + s_{\vec{p}}} \right)$$

Notice that  $U(p_1, p_2, 0)$  is the limit of  $U(\vec{p})$  when  $p_3 \rightarrow 0$ , i.e.

$$U(p_1, p_2, 0) = \frac{1}{\sqrt{2}} \begin{pmatrix} e^{i\delta_{p_3=0}} & 0 \\ 0 & e^{-i\delta_{p_3=0}} \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}. \quad (3.5)$$

Under these rotations the kinetic term remains invariant while the boundary matrices, like the bulk mass, transform covariantly.

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<sup>5</sup>Notice that this symmetry makes sense since superfields  $N$  and  $N_c$  are uncharged.

<sup>6</sup>Actually the spectrum equation does only depend on  $SU(2)$ -invariants. Geometrically speaking these (global) unitary transformations change the basis where the bulk and boundary matrices are expressed but the relations between them remain unaltered.

In the basis where the bulk mass is Dirac-like we have  $\vec{s}_0 = \vec{s}_\pi = (-s, 0, c)$ . The equations of motion now read

$$\left. \begin{aligned} (\partial_5^2 - \square - M^2) \phi &= 0 \\ (\partial_5^2 - \square - M^2) \phi_c &= 0 \end{aligned} \right\} \quad \text{for bosons,} \quad (3.6)$$

$$\left. \begin{aligned} i \bar{\sigma}^\mu \partial_\mu \psi - \partial_5 \bar{\psi}_c - M \bar{\psi}_c &= 0 \\ i \bar{\sigma}^\mu \partial_\mu \psi_c + \partial_5 \bar{\psi} - M \bar{\psi} &= 0 \end{aligned} \right\} \quad \text{for fermions.} \quad (3.7)$$

### 3.1 BOSONIC SOLUTIONS

For a given mass  $m$  the bosonic fields satisfy the Klein-Gordon equation  $\square \phi_{(c)} = -m^2 \phi_{(c)}$ . Therefore the general solution to the 5D equations of motion is given by

$$\Phi(x, y) = A(x) \cos(\Omega y) + B(x) \sin(\Omega y), \quad (3.8)$$

where  $\Phi = \begin{pmatrix} \phi_c \\ \phi \end{pmatrix}$  and  $\Omega$  has been defined in (2.17). The boundary conditions (2.14) at  $y = 0$  impose the restrictions

$$\begin{aligned} A &= \begin{pmatrix} 1 + c - s \\ 1 - c - s \end{pmatrix} a(x), \\ B &= \frac{cM}{\Omega} \begin{pmatrix} 1 + c - s \\ 1 - c - s \end{pmatrix} a(x) + \begin{pmatrix} -1 + c + s \\ 1 + c - s \end{pmatrix} b(x), \end{aligned}$$

where  $a(x)$  and  $b(x)$  are independent complex functions verifying the above Klein-Gordon equation. Finally the boundary conditions at  $y = \pi R$  impose

$$b(x) \sin(\Omega \pi R) = 0, \quad (3.9)$$

$$a(x) \left[ \Omega + \frac{c^2 M^2}{\Omega} \right] \sin(\Omega \pi R) = 0, \quad (3.10)$$

with two possible solutions

1.  $\sin(\Omega \pi R) \neq 0$  and hence  $b(x) = 0$  and  $\Omega^2 = -c^2 M^2$ , whose eigenfunction is

$$\begin{aligned} \Phi^0 &= \begin{pmatrix} 1 + c - s \\ 1 - c - s \end{pmatrix} \left[ \cos(\Omega R y) - \frac{cM}{\Omega} \sin(\Omega R y) \right] \varphi(x) \\ &= \begin{pmatrix} 1 + c - s \\ 1 - c - s \end{pmatrix} e^{-McRy} \varphi(x), \end{aligned} \quad (3.11)$$



2.  $\Omega R = n \in \mathbb{Z}_+$ , with eigenfunction

$$\begin{aligned}\Phi^n &= \begin{pmatrix} 1+c-s \\ 1-c-s \end{pmatrix} f^n(y) \varphi_1^n(x) \\ &+ \begin{pmatrix} -1+c+s \\ 1+c-s \end{pmatrix} g^n(y) \varphi_2^n(x)\end{aligned}\quad (3.12)$$

with  $f^n(y) = \cos(\frac{n}{R}y) - \frac{McR}{n}\sin(\frac{n}{R}y)$  and  $g^n(y) = \sin(\frac{n}{R}y)$ .

### 3.2 FERMIONIC SOLUTIONS

Eq. (3.7) can be re-casted into a single Dirac equation as

$$(\mathrm{i} \gamma^\mu \partial_\mu - \gamma^5 \partial_5 - M) \Psi = 0, \quad (3.13)$$

with  $\Psi$  the Dirac spinor defined in Eq. (2.6), and whose formal solution is given by

$$\Psi = \left[ \cos(\sqrt{-\square - M^2} y) + \gamma^5 \frac{\mathrm{i} \gamma^\mu \partial_\mu - M}{\sqrt{-\square - M^2}} \sin(\sqrt{-\square - M^2} y) \right] \Theta(x),$$

where  $\Theta(x)$  is the initial value at  $y = 0$  which fulfills the 4D Klein-Gordon equation  $\square \Theta + m^2 \Theta = 0$ . Thus the eigenfunction corresponding to the  $m$ -th mode reads

$$\Psi = \left[ \cos(\Omega y) + \gamma^5 \frac{\mathrm{i} \gamma^\mu \partial_\mu - M}{\Omega} \sin(\Omega y) \right] \Theta(x). \quad (3.14)$$

Its initial value satisfies the boundary condition (2.15) at  $y = 0$ , that is:

$$\Theta = \begin{pmatrix} (1+c-s)\chi \\ (1-c-s)\bar{\chi} \end{pmatrix}, \quad (3.15)$$

with  $\chi$  an arbitrary 4D Weyl spinor. Since we are dealing with a 4D Dirac spinor, if  $\xi$  is the Dirac partner of  $\chi$ <sup>7</sup> satisfying the equations of motion

$$\mathrm{i} \sigma^\mu \partial_\mu \bar{\xi} = m \chi, \quad \mathrm{i} \bar{\sigma}^\mu \partial_\mu \chi = m \bar{\xi}, \quad (3.16)$$

by plugging now (3.16) and (3.15) in (3.14) we obtain the fermionic wave eigenfunction

$$\begin{aligned}\Psi &= \begin{pmatrix} (1+c-s)f_-^\Omega(y) \chi \\ (1-c-s)f_+^\Omega(y) \bar{\chi} \end{pmatrix} \\ &+ m \begin{pmatrix} (1-c-s)\xi \\ -(1+c-s)\bar{\xi} \end{pmatrix} \frac{\sin(\Omega y)}{\Omega},\end{aligned}\quad (3.17)$$

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<sup>7</sup>This is in straight analogy to the orbifold case where we have a Dirac spinor in the bulk although parity assignment projects out one of the components at the fixed points such that there we have a single Weyl (Majorana) spinor.

where

$$f_{\pm}^{\Omega}(y) = \cos(\Omega y) \pm \frac{M}{\Omega} \sin(\Omega y). \quad (3.18)$$

Finally the boundary condition at  $y = \pi R$  imposes the further condition

$$\frac{\sin(\Omega \pi R)}{\Omega} \begin{pmatrix} 1-c & s \\ s & 1+c \end{pmatrix} \cdot \begin{pmatrix} -M(1+c-s) & m(1-c-s) \\ M(1-c-s) & -m(1+c-s) \end{pmatrix} \cdot \begin{pmatrix} \chi \\ \xi \end{pmatrix} = 0,$$

which is equivalent to

$$\frac{\sin(\Omega \pi R)}{\Omega} \begin{pmatrix} sM & m \\ sM & m \end{pmatrix} \cdot \begin{pmatrix} \chi \\ \xi \end{pmatrix} = 0. \quad (3.19)$$

Eq. (3.19) again has two possible solutions:

1.  $\Omega R \notin \mathbb{Z}$  with the solution

$$m\xi + sM\chi = 0. \quad (3.20)$$

The condition (3.16) implies now  $\xi = \chi \equiv \eta$  which corresponds to a Majorana spinor and

$$m = -sM. \quad (3.21)$$

2.  $\Omega R = n$  with  $n \in \mathbb{Z}_+$ . In this case we have two independent spinorial degrees of freedom,  $\xi^n$  and  $\chi^n$ , degenerated in mass.

Thus we re-encounter the spectrum (2.18) and the corresponding wave functions turn out to be

$$\begin{aligned} \Psi^n &= \begin{pmatrix} (1+c-s)f_-^n(y) \chi^n \\ (1-c-s)f_+^n(y) \bar{\chi}^n \end{pmatrix} \\ &+ \sqrt{n^2 + M^2 R^2} \begin{pmatrix} (1-c-s) \xi^n \\ -(1+c-s) \bar{\xi}^n \end{pmatrix} \frac{\sin(\frac{n}{R}y)}{n}, \end{aligned} \quad (3.22)$$

$$\Psi^0 = \begin{pmatrix} (1+c-s)\eta \\ (1-c-s)\bar{\eta} \end{pmatrix} e^{-cMy}. \quad (3.23)$$

Some comments about these solutions are now in order. On the one hand notice that they are not, in general, factorizable as  $f(y)g(x)$  which makes it explicit that the orbifold-like

*ansatz* is not suitable <sup>8</sup>. On the other hand we want to remark that the presence of  $\xi$  does not appear as a unitarity problem. From (3.16) it can be expressed in terms of  $\bar{\sigma}^\mu \partial_\mu \chi$  which is in agreement with the uniqueness of the solution to (3.13). In fact  $\xi$  and  $\chi$  can be thought of as off-shell independent degrees of freedom. Finally from (3.14) and (3.6) one immediately checks that given a value of  $\Omega$  the solutions corresponding to  $\pm\Omega$  are the same since there is no associated degeneracy. The sign of the root is a matter of convention since it can be absorbed by the redefinition  $y \rightarrow \pi R - y$ , which shows that both signs do correspond to the same eigenstate.

From now on we will concentrate on the fermionic sector describing RH neutrinos. In particular we will write down the effective 4D action for the hyperfermions  $(\psi, \psi_c)$  coupled to the SM sector which is localized at one of the boundaries, i.e.  $y = y_f$  ( $f = 0, \pi$ ). The Yukawa couplings will induce a Dirac mass connecting the left- and the right-handed neutrinos and we will find the eigenvalues (Dirac or Majorana) of the effective mass matrix in an extra dimensional generalization of the 4D see-saw mechanism <sup>9</sup>. In addition we will discuss in detail the possibility of getting an ultra light mass eigenvalue with natural values of the 5D Yukawa couplings.

## 4 EFFECTIVE ACTION FOR NEUTRINOS

In this section we will develop the 4D effective action for RH neutrinos coupled to the SM matter localized at the brane  $y = y_{f_0}$ . We will obtain the lowest mass eigenvalue in the neutrino sector by solving the characteristic polynomial of the infinite dimensional effective mass matrix involving the LH neutrino and the zero and non-zero modes of RH neutrinos in the background Higgs field. As we will show this can alternatively be done through the effective mass matrix for the LH neutrino and the zero mode of the RH neutrino resulting after integrating out the higher KK modes of RH neutrinos. For the Higgs field which propagates in the bulk and whose zero mode is exponentially localized towards one of the boundaries we will consider the action computed in Ref [30].

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<sup>8</sup>In fact only in the particular case where  $s = 0$ ,  $c = \pm 1$  the solutions (3.22) break up into two orthogonal and factorizable functions. Furthermore in the  $M \rightarrow 0$  limit both functions have a definite parity since the basis of mass eigenfunctions is in that case  $\{\cos(ny/R), \sin(ny/R)\}$ .

<sup>9</sup>See Ref. [3] for a previous non-supersymmetric analysis.

The 5D action under study is <sup>10</sup>

$$\begin{aligned}
\mathcal{S}_{\text{eff}} &= \frac{1}{2} \int_{\Sigma} i\bar{\psi}_c \bar{\sigma}^{\mu} \partial_{\mu} \psi_c + i\psi_c \sigma^{\mu} \partial_{\mu} \bar{\psi}_c + i\bar{\psi} \bar{\sigma}^{\mu} \partial_{\mu} \psi + i\psi \sigma^{\mu} \partial_{\mu} \bar{\psi} \\
&- \frac{1}{2} \int_{\Sigma} \psi_c (-\partial_5 \psi + M\psi) + \psi (\partial_5 \psi_c + M\psi_c) + \text{h.c.} \\
&- \frac{1}{4} \int_{\partial\Sigma} \psi [s_- \psi + (s_3 - 1)\psi_c] + \psi_c [(1 + s_3)\psi - s_+ \psi_c] + \text{h.c.} \\
&+ \int_{y=y_{f_0}} Y_{\nu} \psi_c \nu_L H_c + \text{h.c.}
\end{aligned} \tag{4.1}$$

where  $\nu_L$  denotes the LH neutrino,  $Y_{\nu}$  is the 5D Yukawa coupling (with mass dimension -1),  $y_{f_0}$  stands for the boundary where  $\nu_L$  is localized, i.e.  $f_0 = 0$  or  $f_0 = \pi$  and  $H_c$  is the lowest mode of the Higgs which according to Ref. [30] is given by  $H_c \simeq \sqrt{2M_H} e^{-M_H y} h(x)$  <sup>11</sup>. We now expand  $\psi$  and  $\psi_c$  as

$$\begin{aligned}
\psi_c &= (1 + c - s) e^{-Mcy} \eta(x) \\
&+ \sum_{n \geq 1} \left[ (1 + c - s) f_-^n(y) \chi^n(x) + \frac{m_n}{n} (1 - c - s) \sin(ny/R) \xi^n(x) \right],
\end{aligned}$$

$$\begin{aligned}
\psi &= (1 - c - s) e^{-Mcy} \eta(x) \\
&+ \sum_{n \geq 1} \left[ (1 - c - s) f_+^n(y) \chi^n(x) - \frac{m_n}{n} (1 + c - s) \sin(ny/R) \xi^n(x) \right],
\end{aligned}$$

and whose components satisfy the (free) equations of motion

$$i\bar{\sigma}^{\mu} \partial_{\mu} \eta = -s M \bar{\eta}, \tag{4.2}$$

$$i\bar{\sigma}^{\mu} \partial_{\mu} \begin{pmatrix} \chi^n \\ \xi^n \end{pmatrix} = \frac{1}{R} m_n \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} \bar{\chi}^n \\ \bar{\xi}^n \end{pmatrix}, \tag{4.3}$$

with  $m_n = \sqrt{M^2 R^2 + n^2}$ .

---

<sup>10</sup>The Yukawa interaction terms are strictly localized on the boundaries since they are not  $SU(2)_R$  invariant.

<sup>11</sup>The theory of the Higgs hyperscalar ( $H, H_c$ ) was worked out in detail in Ref. [30]. The bulk mass term in the Higgs sector  $M_H$  is further restricted by the  $SU(2) \otimes U(1)$  invariance of the SM and there is a similar parameter to  $c_{0,\pi}$  defined in section 2,  $c_H$ , which is restricted by the condition of the Higgsino mass  $\mu$  to be  $s_H = \mu/M_H$  so that  $|c_H| \simeq 1$ .

By integrating over the fifth coordinate one obtains the effective 4D action

$$\begin{aligned}
S_{\text{eff}} = & \int d^4x (k_0)^{-2} \left[ \frac{i}{2} \bar{\eta} \bar{\sigma}^\mu \partial_\mu \eta + \frac{1}{2} s M \eta^2 \right] \\
& + \sum_{n \geq 1} \int d^4x \left[ \frac{i}{2} \bar{f}_n k_n \bar{\sigma}^\mu \partial_\mu f_n - \frac{1}{2R} m_n f_n^T k_n \sigma_1 f_n \right] \\
& + \int d^4x Y_\nu \sqrt{2M_H} \mathbf{e}_{f_h} (1 + c - s) \eta \nu_L h \\
& + \sum_{n \geq 1} \int d^4x Y_\nu \sqrt{2M_H} \mathbf{e}_{f_h} (1 + c - s) \chi_n \nu_L h + \text{h.c.} ,
\end{aligned} \tag{4.4}$$

where  $k_0$  is the real number

$$(k_0)^{-2} = \frac{2(1-s)}{cM} (1 - e^{-2cM\pi R}) , \tag{4.5}$$

$k_n$  the tower of matrices

$$k_n = 2\pi R \frac{(m_n)^2}{n^2} (1-s) \left[ 1 + s \frac{MR}{m_n} \sigma_1 \right] , \tag{4.6}$$

$f_n$  stands for

$$f_n = \begin{pmatrix} \chi_n \\ \xi_n \end{pmatrix} ,$$

and

$$\mathbf{e}_{f_h} = e^{-M_H |y_{f_h} - y_{f_0}|} . \tag{4.7}$$

where  $f_h = 0$  or  $f_h = \pi$  depending on the boundary the  $H_c$  zero mode is localized towards. By redefining the modes as

$$\psi_\pm^n = \frac{1}{\sqrt{2} k_\pm^{(n)}} (\chi^n \pm \xi^n) , \tag{4.8}$$

$$\zeta = \frac{1}{k_0} \eta , \tag{4.9}$$

with

$$(k_\pm^{(n)})^{-2} = 2\pi R \frac{(m_n)^2}{n^2} (1-s) \left[ 1 \pm s \frac{MR}{m_n} \right] , \tag{4.10}$$

the 4D effective action (4.4) can be rewritten as

$$\begin{aligned}
S_{\text{eff}} = & \int d^4x \left[ \frac{i}{2} \bar{\zeta} \bar{\sigma}^\mu \partial_\mu \zeta + \frac{1}{2} s M \zeta^2 \right] + \sum_{n \geq 1} \int d^4x \frac{i}{2} (\bar{\psi}_-^n \bar{\sigma}^\mu \partial_\mu \psi_-^n + \bar{\psi}_+^n \bar{\sigma}^\mu \partial_\mu \psi_+^n) \\
& - \sum_{n \geq 1} \int d^4x \frac{1}{2R} m_n \left[ (\psi_+^n)^2 - (\psi_-^n)^2 \right] + \int d^4x Y^{(0)} \zeta \nu_L h \\
& + \sum_{n \geq 1} \int d^4x \left[ Y_-^{(n)} \psi_-^n \nu_L h + Y_+^{(n)} \psi_+^n \nu_L h \right] + \text{h.c.} ,
\end{aligned} \tag{4.11}$$

where

$$Y^{(0)} = Y_\nu (1 + c - s) \sqrt{\frac{|c M M_H|}{1 - s}} \mathbf{e}_{f_h} \mathbf{e}_{f_\nu} \tag{4.12}$$

and

$$Y_\pm^{(n)} = \frac{1}{\sqrt{2}} Y_\nu (1 + c - s) M_H \mathbf{e}_{f_h} k_\pm^{(n)} , \tag{4.13}$$

are the 4D effective Yukawa coupling constants. Here we have defined  $\mathbf{e}_{f_\nu}$  as

$$\mathbf{e}_{f_\nu} = e^{-|c M (y_{f_\nu} - y_{f_0})|} , \tag{4.14}$$

where  $f_\nu$  depends on where the lowest mode of the RH neutrino localizes towards. Once the Higgs gets its vacuum expectation value,  $\langle h \rangle = v$ , the Yukawa couplings turn into Dirac mass terms and we are thus left with an effective mass matrix connecting LH and RH neutrinos given by

$$\begin{aligned}
\mathcal{L}_m = & \frac{1}{2} s M \zeta^2 + Y^{(0)} v \zeta \nu_L - \sum_{n \geq 1} \frac{1}{2R} m_n \left[ (\psi_+^n)^2 - (\psi_-^n)^2 \right] \\
& + \sum_{n \geq 1} v \left\{ Y_+^{(n)} \psi_+^n \nu_L + Y_-^{(n)} \psi_-^n \nu_L \right\} .
\end{aligned} \tag{4.15}$$

Notice that higher modes of RH neutrinos appear in (4.11) as Majorana spinors albeit we started with Dirac fermions. However if we redefine the fields for  $n \neq 0$  as

$$\varphi_+^n = \psi_-^n + \psi_+^n , \tag{4.16}$$

$$\varphi_-^n = \psi_-^n - \psi_+^n , \tag{4.17}$$

we recover the expected Dirac spinors due to the degeneracy in mass of the higher modes.

Now we can find the eigenvalues of the infinite mass matrix (4.15) by computing its characteristic polynomial [3]

$$P(\lambda) = \det \begin{pmatrix} -\lambda & m_D^{(0)} & \dots & m_{D+}^{(n)} & m_{D-}^{(n)} & \dots \\ m_D^{(0)} & sM - \lambda & & & & \\ \vdots & & \ddots & & & \\ m_{D+}^{(n)} & & & -m_n - \lambda & & \\ m_{D-}^{(n)} & & & & m_n - \lambda & \\ \vdots & & & & & \ddots \end{pmatrix}. \quad (4.18)$$

where  $m_D^{(0)} = vY^{(0)}$  and  $m_{D\pm}^{(n)} = vY_{\pm}^{(n)}$ . The determinant of (4.18) yields

$$P(\lambda) = \left[ (\lambda - sM) \prod_{k \geq 1} (\lambda^2 - m_k^2) \right] \left\{ \lambda + \frac{(m_D^{(0)})^2}{sM - \lambda} + \sum_{n \geq 1} \left[ -\frac{(m_{D+}^{(n)})^2}{m_n + \lambda} + \frac{(m_{D-}^{(n)})^2}{m_n - \lambda} \right] \right\} \quad (4.19)$$

When  $s \neq 0$  the smallest eigenvalue,  $\lambda_L$ , will not be that canceling either  $\lambda - sM$  or  $\lambda^2 - m_k^2$ . Therefore it should verify the equation

$$\lambda_L + \frac{(m_D^{(0)})^2}{sM - \lambda_L} + \sum_{n \geq 1} \left[ \frac{(m_{D-}^{(n)})^2}{m_n - \lambda_L} - \frac{(m_{D+}^{(n)})^2}{m_n + \lambda_L} \right] = 0. \quad (4.20)$$

Since  $m_{D\pm}^{(n)}$  and  $m_D^{(0)}$  are  $\sim vY_{\nu}/R$ , and we are assuming this parameter to be much smaller than  $M$ , we can expand the solution for the dimensionless parameter  $\lambda_L/M$  in powers of  $\beta = \frac{vY_{\nu}}{MR}$  as

$$\frac{\lambda_L}{M} = \sum_{\ell=1}^{\infty} \lambda_{2\ell} \beta^{2\ell}. \quad (4.21)$$

which makes sense whenever the lowest order is small. Substituting back in (4.20) we find that at lowest order  $\lambda_L$  is given by

$$\begin{aligned} \frac{\lambda_L}{M} + \frac{(m_D^{(0)})^2}{sM^2} &= - \sum_{n \geq 1} \frac{R}{Mm_n} \left[ (m_{D-}^{(n)})^2 - (m_{D+}^{(n)})^2 \right] \\ &= - \frac{2(1+c)s (\mathbf{e}_{f_h})^2 RM_H v^2 Y_{\nu}^2}{\pi} \sum_{n \geq 1} \frac{n^2}{(n^2 + M^2 R^2)(n^2 + c^2 M^2 R^2)} \end{aligned} \quad (4.22)$$

Finally the series in (4.22) can be computed analytically by means of a Poisson re-summation giving

$$\lambda_L = -v^2 Y_\nu^2 \frac{1+c}{s} M_H (\mathfrak{e}_{f_h})^2 \left[ 2c (\mathfrak{e}_{f_\nu})^2 + \coth(\pi M R) - c \coth(\pi c M R) \right]. \quad (4.23)$$

Notice that the series in (4.22) is finite because both  $m_{D\pm}^{(n)}$  converge to the same limit when  $n \rightarrow \infty$ . This feature is just a consequence of the structure of the bulk mass matrix in (2.3) which is consistent with 5D supersymmetry and Lorentz invariance.

Alternatively this result can be obtained as the diagonalization of the effective action induced after integrating out the higher KK-modes  $\psi_\pm^n$  in (4.15) for momenta much smaller than their masses (i.e. neglecting their kinetic terms). From (4.15) the equations of motion for  $\psi_\pm^n$  are given by

$$\psi_\pm^n = \mp \frac{R m_{D\pm}^{(n)}}{m_n} \nu_L \quad (4.24)$$

and substituting back in (4.15) we find, in matrix form, the following effective mass coupling for  $\nu_L$  and  $\zeta \equiv \nu_R$

$$\frac{1}{2} (\nu_L, \nu_R) \cdot \begin{pmatrix} \mu & m_D^{(0)} \\ m_D^{(0)} & s M \end{pmatrix} \cdot \begin{pmatrix} \nu_L \\ \nu_R \end{pmatrix}, \quad (4.25)$$

with

$$\mu = \sum_{n \geq 1} \frac{R}{m_n} \left[ \left( m_{D+}^{(n)} \right)^2 - \left( m_{D-}^{(n)} \right)^2 \right], \quad (4.26)$$

Since  $\mu$  and  $m_D^{(0)}$  are much smaller than  $M$  the light and heavy eigenvalues  $\lambda_{L,H}$  of the mass matrix in (4.25) are

$$\lambda_L \simeq \mu - \frac{\left( m_D^{(0)} \right)^2}{s M}, \quad \lambda_H \simeq s M. \quad (4.27)$$

where the light eigenvalue  $\lambda_L$  coincides with that found in Eq. (4.23).

A particularly interesting case is found when  $s = 0$ , that is when the boundary matrices are precisely aligned with the bulk mass matrix. In that case the characteristic polynomial simplifies to

$$P_0(\lambda) = \left[ \prod_{k \geq 1} (\lambda^2 - m_k^2) \right] \left\{ \lambda^2 \left( 1 + 2 \sum_{n \geq 1} \frac{\left( m_D^{(n)} \right)^2}{m_n^2 - \lambda^2} \right) - \left( m_D^{(0)} \right)^2 \right\} \quad (4.28)$$



where

$$\left(m_D^{(n)}\right)^2 = 2\beta^2 M^2 M_H R \frac{n^2}{\pi(n^2 + M^2 R^2)}, \quad (4.29)$$

Notice that (4.28) is an equation for  $\lambda^2$  which means that both  $\pm\lambda$  are solutions and thus the set of eigenstates of the whole mass matrix are exactly degenerate by pairs and therefore they can be gathered to yield Dirac fermions. Following the same methods used above we find that the lowest eigenvalue is given by

$$\lambda_{L\pm} = \pm 2 Y_\nu v \mathfrak{e}_{f_h} \sqrt{M_H M} e^{-\pi M y_{f_\nu}}, \quad (4.30)$$

The effective mass matrix for  $\nu_L, \nu_R$  will be given in that case by

$$\begin{pmatrix} 0 & m_D \\ m_D & 0 \end{pmatrix}. \quad (4.31)$$

where we have defined  $m_D = 2 Y_\nu v \mathfrak{e}_{f_h} \sqrt{M_H M} e^{-\pi M y_{f_\nu}}$ .

The degeneracy of the spectrum for  $s = 0$  can be understood in terms of a symmetry which takes place only within this case. As a matter of fact  $s = 0$  means that the vectors  $\vec{p}, \vec{s}_0, \vec{s}_\pi$  are all aligned along the same direction and hence a  $U(1)$  subgroup of unitary rotations around this direction leaves the action invariant. In terms of the fermion components these transformations translate into

$$(\eta, \chi^n) \rightarrow e^{i\alpha} (\eta, \chi^n), \quad \xi^n \rightarrow e^{-i\alpha} \xi^n, \quad \nu_L \rightarrow e^{-i\alpha} \nu_L \quad (4.32)$$

where  $\alpha$  is a real parameter. Notice that this symmetry forbids any Majorana mass term<sup>12</sup>, in particular for  $\nu_L$ , and hence  $\mu$  must vanish, as it is evident from Eq. (4.4).

## 5 DISCUSSION ON NEUTRINO MASSES

In this section we will apply the previous results to discuss the possibility of getting, within this kind of models, an (ultralight) neutrino mass in the sub meV range. The first task will be to set the range of dimensional Yukawa couplings which appear in the 5D action in the leptonic sector

$$\int_{\partial\Sigma} (Y_{\nu_\ell} H_c \nu_L \psi_c + Y_\ell H \ell_L e_R) \quad (5.1)$$

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<sup>12</sup>This symmetry plays the role of the lepton number symmetry of the SM.

i.e.  $Y_{\nu_\ell}$ , with mass dimension -1, and  $Y_\ell$ , with mass dimension -1/2, where  $\ell = \{\tau, \mu, e\}$ . A naive estimate of wave function renormalization corrections to the Yukawa couplings in the 5D theory sets bounds as

$$y_{\nu_\ell} \equiv \frac{Y_{\nu_\ell}}{R} \lesssim \frac{4\pi}{\Lambda R}, \quad y_\ell \equiv \frac{Y_\ell}{\sqrt{R}} \lesssim \frac{4\pi}{\sqrt{\Lambda R}} \quad (5.2)$$

so that taking  $\Lambda R \sim 20$  we obtain  $\mathcal{O}(1)$  upper bounds on the dimensionless Yukawa couplings  $y_{\nu_\ell, \ell}$ . We can now distinguish two different scenarios where neutrino masses are either Dirac or Majorana:

## 5.1 DIRAC MASS

We will assume here that all the SM matter is strictly localized on the  $y = 0$  brane and the zero mode of the Higgs is localized towards it as well, thus  $\epsilon_{f_h} = 1$ , while the zero mode of RH neutrino is exponentially localized towards  $y = \pi R$ , i.e.  $y_{f_\nu} = \pi R$  as Fig. 1 shows.

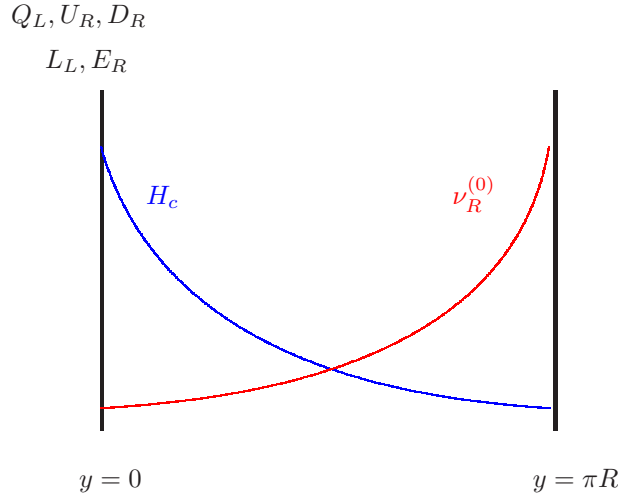


Figure 1: Bulk and brane matter distribution for a neutrino Dirac-like mass.

We will consider for the moment the particular case where  $s = 0$ , i.e. where vectors  $\vec{p}$  and  $\vec{s}_{0,\pi}$  are all aligned along the same direction. As it was shown in the previous section we obtain a Dirac mass connecting  $\nu_L$  and  $\nu_R$ , which is of order  $m_{\nu_\ell}^D \sim 2v Y_{\nu_\ell} \sqrt{M M_H} \epsilon_R$ , with

$\epsilon_R = e^{-M\pi R}$ . In Fig. 2 we show the Dirac mass as a function of  $MR$  for  $1/R = 5$  TeV,  $y_{\nu_\ell} \sim 1$  and  $M_H R \sim 1.6$  [30]<sup>13</sup>. We can see from Fig. 2 that  $m_{\nu_\ell}^D \lesssim 1$  eV for  $MR \gtrsim 9$  although  $m_{\nu_\ell}^D$  decreases exponentially when  $MR$  increases and thus  $m_{\nu_\ell}^D \simeq 1$  meV for  $MR \simeq 11$ . In this scenario there is no wave function suppression for the charged leptons whose Yukawa couplings should therefore be given by  $y_\ell \sim m_\ell/v$ .

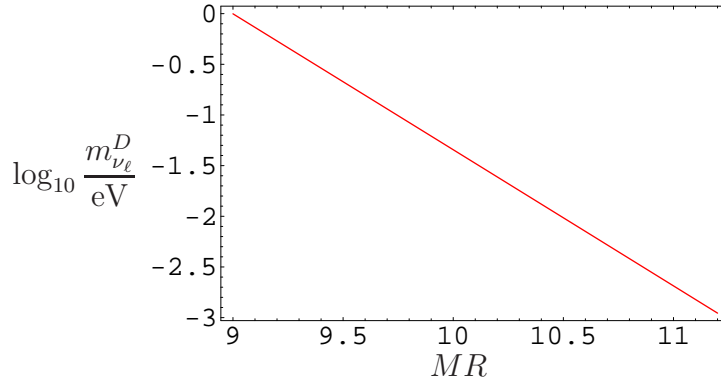


Figure 2:  $m_{\nu_\ell}^D$  as a function of  $MR$  for  $M_H R = 1.6$  and  $s = 0$ .

If  $s \neq 0$  in this scenario the lowest eigenvalue is a Majorana mass given by

$$m_{\nu_\ell}^M \sim Y_{\nu_\ell}^2 v^2 M_H \left[ 2c e^{-2\pi|cM|R} + \coth(\pi MR) - c \coth(\pi c MR) \right], \quad (5.3)$$

which in general will be too large, unless there is a strong suppression of the 5D Yukawa constants as  $y_{\nu_\ell} \sim 10^{-6}$ , which is similar to the electron Yukawa coupling in this kind of models  $y_e$ .

Yet another possibility could be to localize the lowest mode of the RH neutrino towards  $y = 0$ , corresponding to  $cM > 0$ . Considering now  $MR \gg 1$  the mass eigenvalue is proportional to  $(c + \text{sign}(M) + 2e^{-2\pi|M|R} - 2ce^{-2\pi c MR})$ . Then by choosing  $c = -\text{sign}(M)$  we could be left with an exponentially suppressed Majorana mass. However this value of  $c$  is not consistent with the initial hypothesis  $cM > 0$ . In fact the smallness of the Majorana eigenmass is naturally achieved with a different localization of the quark and lepton sector within the SM as we will see in the next section.

<sup>13</sup>As it is shown in Ref. [30] for values of  $M_H R$  near  $M_H R \sim 1.6$ , when supersymmetry is globally broken by Scherk-Schwarz boundary conditions the spectrum of the Higgs presents a tachyon at the tree level, which partially cancels the positive one-loop radiative correction to the Higgs mass due to the gauge coupling and allows EWSB to take place at the two-loop level with a modest amount of fine tuning.

## 5.2 MAJORANA MASS

The main obstruction to get a small Majorana mass eigenvalue out of the effective mass matrix (4.25) for the  $s \neq 0$  case is that the Yukawa couplings of the higher KK modes are not suppressed if the SM matter is located on the same boundary where the Higgs localizes towards. However by allowing the Standard Model matter to be split into different branes <sup>14</sup>, for instance quarks localized in the same boundary (quark brane) where the Higgs localizes towards, and leptons localized in the opposite boundary (lepton brane) as Fig. 3 shows, then by means of the small exponential wave function factor  $\epsilon_{f_h}$ , the whole tower of effective Yukawa couplings will be exponentially suppressed by the Higgs localization and so  $\mu$  will be.

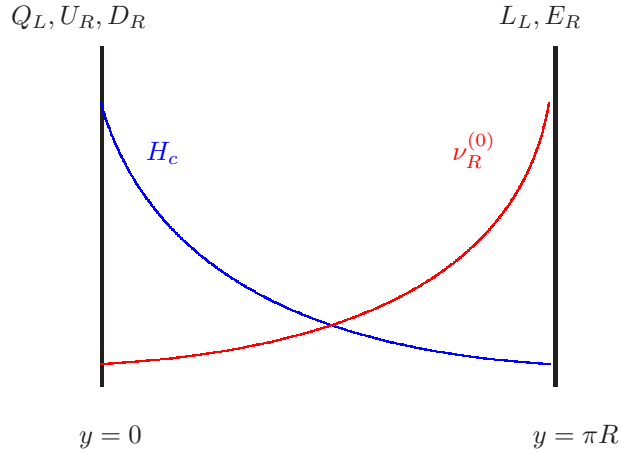


Figure 3: Bulk and brane matter distribution for a neutrino Majorana-like mass. The  $\nu_R$  propagates in the bulk with mass  $M$ .

Now the effective mass matrix is analogous to the previous case except for the global exponential suppression in the Dirac couplings, namely,  $\mu \rightarrow \epsilon_H^2 \mu$  with  $\epsilon_H = e^{-\pi M_H R}$ . In addition we will assume the lowest mode of the RH neutrino to localize towards the leptonic brane, i.e.  $y_{f_\nu} = 0$ ,

<sup>14</sup>Such a splitting may find its justification within the context of intersecting branes in string theory. See e.g. [31] and references therein.

corresponding thus to  $cM < 0$ . We then find that the lowest neutrino Majorana mass eigenvalue is given by

$$m_{\nu_\ell}^M = \epsilon_H^2 v^2 Y_{\nu_\ell}^2 M_H \frac{1+c}{s} [2c + \coth(\pi MR) - c \coth(c\pi MR)] . \quad (5.4)$$

Notice the almost independence on the RH neutrino bulk mass  $M$ . Albeit its presence is absolutely necessary to provide the existence of a lowest Majorana mass eigenvalue <sup>15</sup> it is shielded by the higher RH neutrino modes. In Fig. 4 we plot  $m_{\nu_\ell}^M$  as a function of  $\log_{10} y_{\nu_\ell}$  for fixed values of  $c$  and  $MR$ .

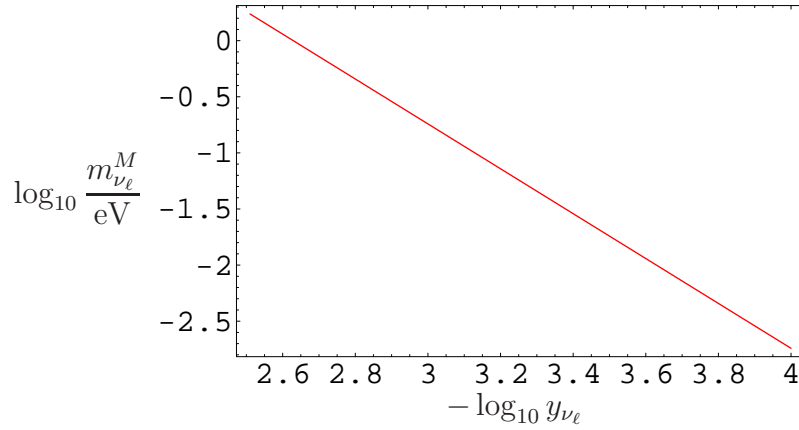


Figure 4: Majorana neutrino mass,  $m_{\nu_\ell}^M$ , as a function of  $-\log_{10} y_{\nu_\ell}$  for  $c = -1/2$ ,  $M_H R = 1.6$  and  $MR = 5$ .

From Fig. 4 we can see that generically  $m_{\nu_\ell}^M \lesssim 1$  eV implies  $y_{\nu_\ell} \lesssim 10^{-3}$ .

The charged leptons, on the other hand, have masses

$$m_\ell \sim v y_\ell \sqrt{M_H R} \epsilon_H , \quad (5.5)$$

By fixing in this scenario the Higgs localizing mass to its previous value  $M_H R = 1.6$  we can predict the correct value of the  $\tau$  mass [32] by means of a 5D Yukawa coupling  $y_\tau \simeq 1$  while  $y_\ell \simeq m_\ell/m_\tau$  for the first two generations ( $\ell = e, \mu$ ).

An interesting particular case arises here. Given that  $cM$  is negative, in the limit when  $|cMR| \gg 1$  Eq. (5.4) reads

$$m_{\nu_\ell}^M \sim Y_{\nu_\ell}^2 v^2 \epsilon_H^2 M_H \frac{1+c}{s} [3c + \text{sign}(M) + 2\text{sign}(M) e^{-2\pi|M|R} + 2c e^{-2\pi|cM|R}] . \quad (5.6)$$

---

<sup>15</sup>In case of vanishing  $M$  we would be left with a lowest Dirac eigenvalue or, at most, with two almost degenerate Majorana eigenstates.

Considering now the particular value  $c = -\frac{1}{3}\text{sign}(M)$  we find a doubly suppressed Majorana mass eigenvalue. For instance for the case  $M > 0$  and  $c = -1/3$  one gets

$$m_{\nu_\ell}^M \sim \frac{\sqrt{2}}{3} Y_{\nu_\ell}^2 v^2 \epsilon_H^2 e^{-\frac{2}{3}\pi MR}. \quad (5.7)$$

which has a doubly suppressed exponential behavior both from  $M_H R$  and  $MR$ . In that case one can get tiny Majorana neutrino masses from the localization of the zero mode of  $\nu_R$  for  $\mathcal{O}(1)$  values of the 5D Yukawa couplings  $y_{\nu_\ell}$ . This is shown in Fig. 5 where the Majorana mass  $m_{\nu_\ell}^M$  is plotted versus  $|MR|$  for  $y_{\nu_\ell} = 1$  and  $c = \pm 1/3$ .

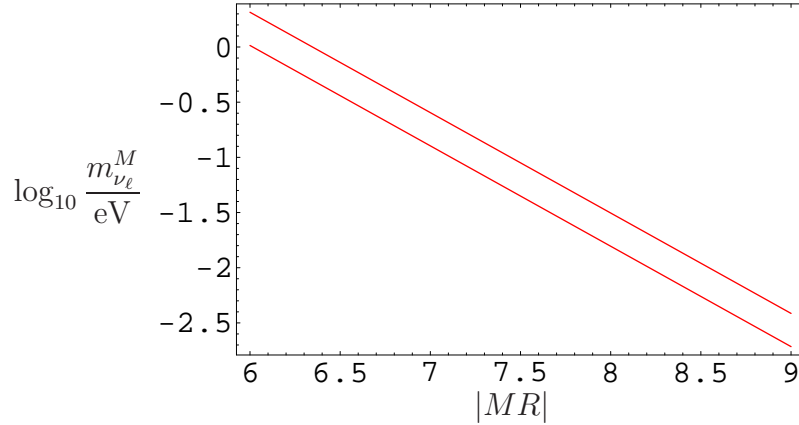


Figure 5: Neutrino Majorana mass,  $m_{\nu_\ell}^M$ , as a function of  $|MR|$  for  $M_H R = 1.6$  and  $c = 1/3$  (upper curve),  $c = -1/3$  (lower curve).

## 6 CONCLUSIONS

In this paper we have investigated the nature and size of the LH neutrino masses in a supersymmetric 5D model compactified in the space  $\mathcal{M}_4 \times I$  where  $I = [0, \pi R]$  is a finite interval of length  $\pi R$  and  $R$  the compactification radius. Quarks and leptons are localized on some of the boundaries and the gauge and Higgs sector propagate in the bulk of the fifth dimension. In this model, as we have found in previous works, supersymmetry can be globally broken by a Scherk-Schwarz twist giving a mass to gauginos (and gravitinos) which is transmitted by one-loop radiative corrections to squarks localized on the boundaries, and provides a very

characteristic pattern for the supersymmetric spectrum which could be easily identified experimentally at LHC whenever it is found [27]. Furthermore electroweak breaking proceeds by radiative corrections [30] providing at low energy an MSSM-like 4D model.

We have found that the nature of the LH neutrino mass depends on the relative orientation of the boundary terms, given by the vector  $\vec{s}$ , with respect to the bulk mass term characterized by the vector  $M\vec{p}$ . In the generic case of arbitrary orientations lepton number is violated and neutrinos are Majorana fermions. In the particular case where vectors  $\vec{s}$  and  $\vec{p}$  are aligned (or anti-aligned) there is an extra global  $U(1)$  symmetry which prevents lepton number breaking and LH neutrinos get a Dirac mass.

As for the size of the LH neutrino masses, in the case of Dirac neutrinos, the smallness of neutrino masses is provided by the bulk mass  $M$  which should localize the zero mode of RH neutrinos towards the opposite boundary to that where SM fermions are localized: sub-eV masses are obtained for  $MR \sim 10$ . In the cases where neutrino masses are Majorana, in order to avoid introducing too small neutrino (dimensional) Yukawa couplings we have to localize quarks and leptons on different boundaries. In that case sub-eV neutrino masses are provided in general for Yukawa couplings  $\sim 10^{-3}R$ . However for particular values of  $\vec{s} \cdot \vec{p}$  there is a cancellation in the neutrino mass matrix in the limit of large localizing masses and the correct values  $\sim \text{meV}$  simply require  $MR \sim 10$ .

Following the lines of our calculation it should be easy to describe textures of RH neutrino masses describing the different patterns for LH neutrino masses and mixings (see e.g. [33]). It should be enough to introduce the corresponding three-by-three mass matrices in the 5D bulk and boundaries and to carry on the parallel calculation. In the cases where Dirac or Majorana neutrino masses are controlled by the localizing masses of the RH neutrinos, since the former depend exponentially on the latter a modest change in the corresponding RH mass eigenvalues should be able to describe realistic neutrino spectra. Of course since the RH neutrino masses are an input in our theory, even if correct spectra do not require to fine-tune any parameters, we should not call this a “solution to the neutrino mass problem” until some more fundamental theory (e.g. string theory) would give us the correct values for the heavy masses. In fact this was not the aim of our work but rather a classification of the different solutions of the 5D theory providing realistic spectra for neutrino masses and yielding hints for possible future discoveries at LHC.

## ACKNOWLEDGMENTS

Work supported in part by the European Commission under the European Union through the Marie Curie Research and Training Networks “Quest for Unification” (MRTN-CT-2004-503369) and “UniverseNet” (MRTN-CT-2006-035863); by the Spanish Consolider-Ingenio 2010 Programme CPAN (CSD2007-00042); and by CICYT, Spain, under contract FPA 2005-02211. We would also like to thank G. von Gersdorff for useful discussions and remarks and for initial collaboration in this work, and to C. Biggio for discussions.

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